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CHOOSING THE TYPE OF GROWTH CURVE IN FORECASTING ECONOMIC DYNAMICS

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ВИБІР ВИДУ КРИВОЇ ЗРОСТАННЯ ДЛЯ ПРОГНОЗУВАННЯ ЕКОНОМІЧНОЇ ДИНАМІКИ

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ВЫБОР ВИДА КРИВОЙ РОСТА ДЛЯ ПРОГНОЗИРОВАНИЯ ЭКОНОМИЧЕСКОЙ ДИНАМИКИ

The article summarizes the theoretical and practical aspects of the methods of preliminary selection of growth curves and the quantitative evaluation of their parameters, systematized and analyzed the types of trend models for economic analysis and extrapolation forecasting of dynamics based on time series.

Key words: *absolute growth; adequate model; predicted indicator; a number of dynamics; development trend; growth rate.*

Fig.: 1. Bibl.: 13.

У статті узагальнено теоретичні та практичні аспекти методів попереднього вибору кривих зростання і кількісного оцінювання їх параметрів, систематизовано та проаналізовано види трендових моделей для економічного аналізу й екстраполяційного прогнозування динаміки на основі часових рядів.

Ключові слова: *абсолютний приріст; адекватна модель; прогнозований показник; ряд динаміки; тенденція розвитку; швидкість зростання.*

Рис.: 1. Бібл.: 13.

В статье обобщены теоретические и практические аспекты методов предварительного выбора кривых роста и количественной оценки их параметров, систематизированы и проанализированы виды трендовых моделей для экономического анализа и экстраполяционного прогнозирования динамики на базе временных рядов.

Ключевые слова: *абсолютный прирост; адекватная модель; прогнозируемый показатель; ряд динамики; тенденция развития; скорость роста.*

Рис.: 1. Библ.: 13.

JEL Classification: C53

Target setting. Globally established Goals of Sustainable Development [1] are a part of the 2030 Development Agenda, adopted by the world leaders at the historically significant UN Summit, in the framework of the 70th Session of UN General Assembly on September 2015 in New York and are aimed at ensuring the integration of efforts for economic growth, striving for social justice and efficient use of natural resources, which requires deep social and economic transformations in Ukraine and new approaches to the possibility of global partnership. The objectives of applying adequate and up-to-date models for economic analysis and forecasting in terms of promoting a development-oriented policy are still relevant.

Actual scientific researches and issues analysis. In the works by V.M. Geits [2], Ye.M. Libanova [3], O.I. Chernyak [4], I.H. Lukyanenko [5], A.V. Matviichuk [6], A.V. Holovach [7], G.L. Gromyko [8], A.M. Yerina [9] and other national and foreign scientists, much attention has been paid to the study of dynamics of economic and social processes.

Uninvestigated parts of general matters defining. Despite the availability of a sufficient number of research works on the question of analysis of development patterns, this issue needs further development in the direction of working out the trend models, which can be most relevant in modern conditions of significant destabilizing impact of constant changes in the environment of functioning economies, as well as recommendations on theoretical and practical aspects of the choice methods, including those methods that take into account specifics of modern Ukrainian economy.

Purpose of the article. The main purpose of this work is to systematize the types of growth curves for modelling and forecasting economic processes and to carry out a comparative analysis of conditions of their use, together with the generalization of theoretical and practical aspects of preliminary choice methods of growth curves and quantitative evaluation of their parameters.

The statement of basic materials. Trend is a stable systematic change that determines the general direction of development, the main tendency of the process for a long time. In this context, the economical and mathematical dynamic model [10], in which development of a simulated economic system is reflected through the trend of its main indicators, is called the trend model. The main purpose of creating the trend models of dynamics is to perform the forecast on their basis, concerning the development of a studied process or phenomenon for the future time period.

Forecasting based on a number of series of dynamics refers to one-dimensional forecasting methods built on extrapolation, that is continuation of the past trend in the future. It is assumed that, firstly, a predicted indicator is formed under the influence of a large number of factors, which are either impossible to identify, or have no related information; accordingly, the change of this indicator is not related to the factors, but is connected with the passage of time, which is manifested in the appearance of one-dimensional dynamics series. Secondly, the series of dynamics really has a prevailing tendency (trend) and, thirdly, the general conditions determining development of an indicator in the past will remain without significant changes during the period of prediction.

In the short-term forecasting, as well as forecasting in the situation of changing external conditions, when the recent implementation of the investigated process is important, adaptive models of forecasting – the models of discounted data, capable of quickly adapting their structure and parameters to the change of conditions, have proved their efficiency in [11]. When setting their parameters, the levels of dynamics series are assigned different weights, depending on how strong their influence is considered on the current level and allowing to take into account any changes in the trend, as well as any fluctuations in which the pattern is followed. All adaptive models are based upon two schemes: the moving average scheme (the SS models, for example, Brown and Holt model) where the current level is the weighted average of all previous levels, with the weight decreasing according to the distance from the last level, and autoregressive scheme (AR models) where evaluation of the current level is a weighted sum of not all but several previous levels, while the weight coefficients are not ranked, that is, the informational value of the levels of dynamics series is determined not by their proximity to the modelled level, but by the tightness of connection between them.

In case of extrapolating forecasting of dynamics, based on the time series using trend models, the following main stages are performed:

1) preliminary data analysis;
2) forming a set of models, for example, a set of growth curves called as expectant functions;
3) quantitative evaluation of the model parameters;
4) determining the adequacy of models;
5) evaluation of the accuracy of adequate models;
6) the choice of the best model;
7) obtaining the point and interval forecasts;
8) verification of the forecast.

For modelling and forecasting economic processes polynomial, exponential and *S*-shaped growth curves are most often used. Thus, the simplest ones, the polynomial growth curves can be used for approximation and forecasting of the processes, the further development of which does not depend on the achieved level. In general terms:

$\hat{y}_t = b_0 + b_1 \cdot t$ – a polynomial of the first degree,

$\hat{y}_t = b_0 + b_1 \cdot t + b_2 \cdot t^2$ – a polynomial of the second degree,

$\hat{y}_t = b_0 + b_1 \cdot t + b_2 \cdot t^2 + b_3 \cdot t^3$ – a polynomial of the third degree and so on.

$\hat{y}_t$ – theoretical levels of dynamics series, $t=\overline{1;n}$; n – number of observations (levels of dynamics series); a parameter b_1 is called a linear increment, a parameter b_2 is the growth acceleration, a parameter b_3 is the change of growth acceleration.

A polynomial of the first degree is characterized by a constant law of growth: if the first increments are calculated by the formula $\Delta y_t = y_t - y_{t-1}, t=\overline{2;n}$, they will have a constant value equal to b_1.

If the first increments are calculated for a polynomial of the second degree, they will be linearly dependent on time, and a series of the first absolute increments $\Delta y_2, \Delta y_3, ..., \Delta y_n$ in the graph will be represented by a straight line. The second absolute increments $\Delta y_t^{(2)} = \Delta y_t - \Delta y_{t-1}$ for the polynomial of the second degree will be constant (equal to each other).

For a polynomial of the third degree the first increments will form a polynomial of the second degree, the second increments will form a linear function of time and the third increments will be a constant value.

In contrast to polynomials the use of exponential growth curves suggests that the further development will depend on the achieved level, for example, an increment dependent on the function value $\hat{y}_t$.

A simple exponent can be represented by a function

$$\hat{y}_t = a \cdot b^t ,$$

where *a*, *b* are positive numbers; thus, when $b>1$, the function increases over the time *t*, and when $b<1$ the function decreases.

An ordinate of the simple exponent changes with the constant growth rate, the ratio of an absolute growth to the ordinate itself $\frac{\Delta y_t}{y_t} = \frac{y_t - y_{t-1}}{y_t} = 1 - \frac{1}{b}$ is a constant value and moreover, the logarithms of the ordinates ($\log \hat{y}_t = \log a + t \cdot \log b$) are linearly dependent on the time factor *t*.

One of the options of a modified exponent has the form:

$$\hat{y}_t = k + a \cdot b^t ,$$

here *a*, *b* are stable: $a<0$, $0<b<1$;
k is an asymptote of this function, that is the function values are infinitely approaching (from the bottom) to the magnitude *k*.

If to find the logarithm of the first absolute increments of the last function, there will be a function that is linearly time dependent. The ratio of the last two increments $\frac{\Delta y_t}{\Delta y_{t-1}} = \frac{y_t - y_{t-1}}{y_{t-1} - y_{t-2}} = b$ is a constant value.

In economy the fairly common processes are those that are initially slowly developing, then they are accelerating, then slowing down again, approaching a certain limit (for example, a change in demand for goods that have the ability to reach a certain saturation level), which are modelled using the so-called *S*-shaped curves with the Gompertz curve among them (1) applied, for example, for modelling the mortality rates in demographics or describing the dynamics of living standards, and also the Pearl and Reed curve (a logistic curve) (2) that is an increasing function, the growth rate of which at each moment of time is proportional to the achieved level of function, as well as the difference between the limit values *k* and achieved level.

$$\hat{y}_t = k \cdot a^{b^t}, \qquad (1)$$

where a, b are positive parameters, and $b<1$;
k is an asymptote of the function.

$$\hat{y}_t = \frac{k}{1 + a \cdot e^{-b \cdot t}} \text{ or } \hat{y}_t = \frac{k}{1 + a \cdot b^{-t}}, \text{ or } \hat{y}_t = \frac{k}{1 + 10^{a - b \cdot t}}, \qquad (2)$$

here a, b are also positive parameters;
k is a limit value of the function together with the infinite time growth.

The Gompertz curve has got four sections: in the first section an increment in the function is insignificant, in the second section this growth increases, in the third section the increment is almost constant and in the fourth section there is a slowdown in the growth rate and the function is continuously approaching the value k. As a result, the curve configuration resembles the Latin *S*.

The logarithm of the function (1) is an exponential curve, the logarithm of the ratio of the first increment to the very ordinate of the function is a linear function of time.

The logarithm of the ratio of the first increment of the function (2) to the square of its value (an ordinate) is a linear function of time. Configuration of logistic curve graph is approximately the same as the Gompertz curve, but has a point of symmetry that coincides with the point of inflection.

In the preliminary choice for a particular series of dynamics $y_1, y_2, ..., y_n$ of the polynomial curve, when, firstly, the levels of a series consist of only two components: a trend and a random component, and, secondly, when the trend is sufficiently smooth so that it can be approximated by a polynomial of a certain degree, the Tintner method (the finite difference method) has got the largest spread. At the first stage of the implementation of this method the differences (absolute increments) are calculated up to the k-th order inclusive (for approximation of economic processes usually to the fourth order):

$$\Delta y_t = y_t - y_{t-1},$$
$$\Delta y_t^{(2)} = \Delta y_t - \Delta y_{t-1},$$
$$\Delta y_t^{(3)} = \Delta y_t^{(2)} - \Delta y_{t-1}^{(2)},$$
$$\dots$$
$$\Delta y_t^{(k)} = \Delta y_t^{(k-1)} - \Delta y_{t-1}^{(k-1)}.$$

Next, for the initial series of dynamics and for each difference line $(k \in \{1; 2; ...\})$ dispersions are calculated:

$$\sigma_0^2 = \frac{\sum_{t=1}^{n} y_t^2 - \frac{1}{n} \cdot \left(\sum_{t=1}^{n} y_t \right)^2}{n-1}$$ – for the initial series,

$$\sigma_k^2 = \frac{\sum_{t=k+1}^{n} \left(\Delta y_t^{(k)} \right)^2}{(n-k) \cdot C_{2k}^k}$$ – for the series of differences of k-th order (C_{2k}^k is a binomial coefficient).

The comparison of deviations $\left| \sigma_k^2 - \sigma_{k-1}^2 \right|$ of each subsequent dispersion from the previous one is carried out and if this value does not exceed a predetermined positive value for a particular k (the dispersion of one order), the degree of an approximating polynomial must be equal to $k–1$.

A more universal approach of preliminary choice of the growth curves from a wide variety of them is the method of growth characteristics, which is based on the use of certain characteristic properties of the above-discussed curves. In applying this method, the initial series of dynamics is pre-smoothed by the simple moving average method. For example, for a smoothing interval $m = 3$ the aligned levels are calculated by the formula

$$\overline{y}_t = \frac{y_{t-1} + y_t + y_{t+1}}{3},$$

besides, in order not to lose the initial and final levels, they are aligned by the formulas

$$\overline{y}_1 = \frac{5 \cdot y_1 + 2 \cdot y_2 - y_3}{6},$$

$$\overline{y}_n = \frac{-y_{n-2} + 2 \cdot y_{n-1} + 5 \cdot y_n}{6}.$$

After that the first average increments are calculated

$$\overline{\Delta y}_t = \frac{\overline{y}_{t+1} - \overline{y}_{t-1}}{2}, t = \overline{2; n-1},$$

then the second average increments

$$\overline{\Delta y}_t^{(2)} = \frac{\overline{\Delta y}_{t+1} - \overline{\Delta y}_{t-1}}{2},$$

and also a number of derivative values connected with the calculated average increments and smoothed levels of the series:

$$\frac{\overline{\Delta y}_t}{\overline{y}_t}; \quad log\, \overline{\Delta y}_t; \quad log\, \frac{\overline{\Delta y}_t}{\overline{y}_t}; \quad log\, \frac{\overline{\Delta y}_t}{\overline{y}_t^2}.$$

In accordance with the nature of change in average increments and derivative characteristics, a type of the growth curve for the initial series of dynamics is determined (Figure).

In practice, as a rule two or three growth curves are selected for the further research and construction of a trend model of the studied series of dynamics.

Parameters of polynomial curves are estimated by the least squares method, which leads to a system of normal equations for determining unknown parameters of selected curves [12, p. 117–118].

Parameters of exponential and *S*-shaped curves are determined using more sophisticated methods. Thus, for a simple exponent a logarithm of the function is previously taken (by a common logarithm, base logarithm, etc.), resulting in a linear expression:

$$\hat{y}_t = a \cdot b^t \Rightarrow \log\, \hat{y}_t = \log a + t \cdot \log b,$$

after that, for unknown parameters $log\ a$ and $log\ b$ a system of normal equations, based on the least squares method, is formed similar to the system for determination of linear parameters. As a result of solving this system, the logarithms of parameters are determined, and then the very parameters of a model *a* and *b* are identified.

Name of an indicator and/or its designation	Nature of change of the indicator in time	Type of the growth curve
The first average increment $\overline{\Delta y_t}$	Approximately the same	Polynomial of the first degree (linear polynomial)
The first average increment $\overline{\Delta y_t}$	Changing linearly	Polynomial of the second degree (parabolic curve)
The second average increment $\overline{\Delta y_t^{(2)}}$	Changing linearly	Polynomial of the third degree (cubical parabola)
$\frac{\overline{\Delta y_t}}{\bar{y}_t}$	Approximately the same	Simple exponent
$\log \overline{\Delta y_t}$	Changing linearly	Modified exponent
$\log \frac{\overline{\Delta y_t}}{\bar{y}_t}$	Changing linearly	Gompertz curve
$\log \frac{\overline{\Delta y_t}}{\bar{y}_t^2}$	Changing linearly	Pearl and Reed curve

Fig. Data for the preliminary selection of the best growth curve for modelling and forecasting dynamics

In case of determining parameters of the growth curves with asymptotes (a modified exponent, Gompertz curve, logistic curve), two options are distinguished:

1) if the value of the asymptote k is known, the setting of parameters is reduced, by a simple modification of the function with the following logarithm, to the solution of a system of normal equations, the unknown values of which are logarithms of the curve parameters;

2) if the value of the asymptote is not known in advance, approximate methods are used to find the parameters of the growth curves – the three-point method, the method of three sums and others.

To understand how constructed models can correspond to the reality represented by a series of dynamics, how justified is the use of these models for analysis and forecasting of the phenomenon under study, each of them is evaluated as for their adequacy to the real state of things, and after that the most precise model is chosen from the number of adequate models. At present, the only approach to verify the adequacy of models has not been developed yet, though a number of methods proving their efficiency in practice have been determined, one of which involves the consistent verification of four properties of a residual component $\varepsilon_t = y_t - \hat{y}_t \ (t = \overline{1; n})$: the random nature of fluctuations of the residual sequence levels ε_t, the correspondence of the random component distribution to the normal law, the equality of mathematical expectation of the random component to zero and the independence of the random component levels [13, p. 253–258].

Conclusions and propositions. Up to the present moment, an effective approach to evaluating the forecast quality before its implementation has not been invented. An indicator of the forecast value is not only its reliability, but also its efficiency. Even in cases when the forecast has not been confirmed during the verification, the user can at least partially control the course of events, influence the process and apply the forecast information for the desired course of action. Having received a forecast of events with an unwanted direction of perspective development, the user can take steps to make the forecast ineligible (self-destructive forecasting). If the forecast predicts an acceptable course of events, the user can apply all his capabilities to increase the probability of the expected forecast (self-regulating forecasting). Choosing the model type is an important step in the process of constructing and implementing econometric models, thus undoubtedly affecting the quality of forecasting.

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